Fluid Mechanics
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Lecture - 15
Bernoulli's Equation: Problems Solving on Black Board

Good afternoon for this mock course on fluid mechanics. Today have very interesting class on blackboard, solving the problems on Bernoulli's equations applications.

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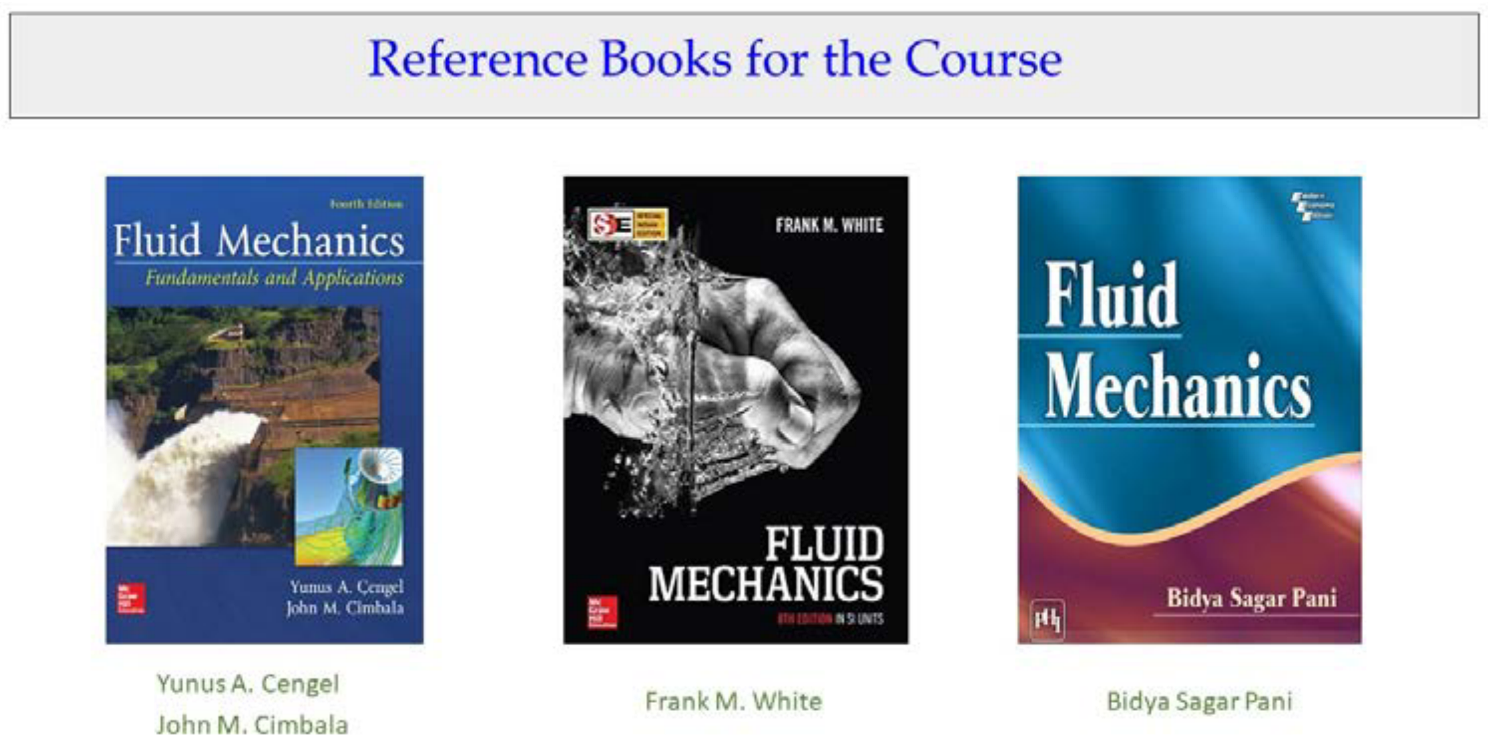

Before starting this class as usual in the next class what we discuss that we are following these three books Cengel, Cimbala, F.M. White and Bidya Sagar Pani. And today we will focus more on solving the GATE exam questions.

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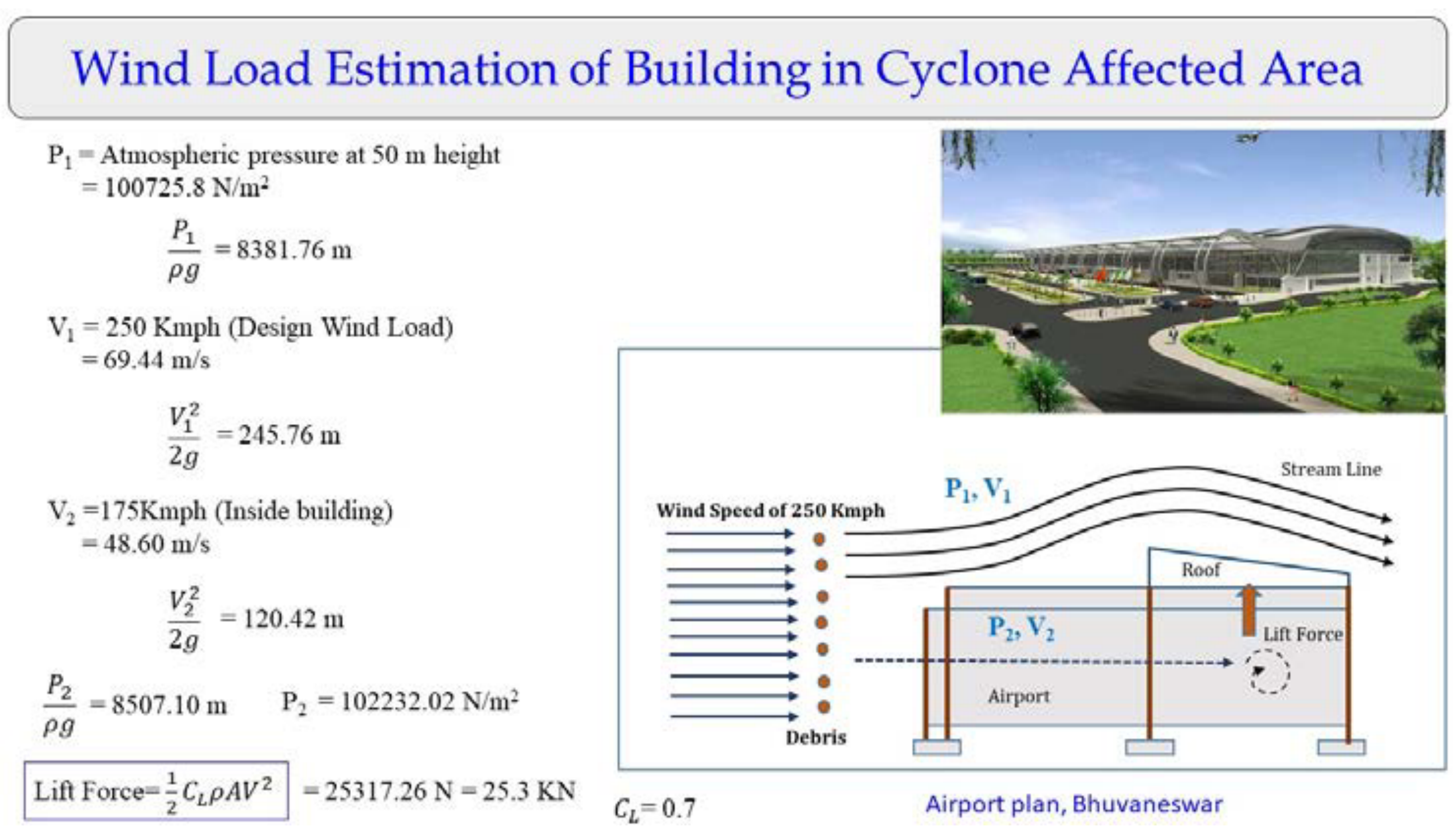
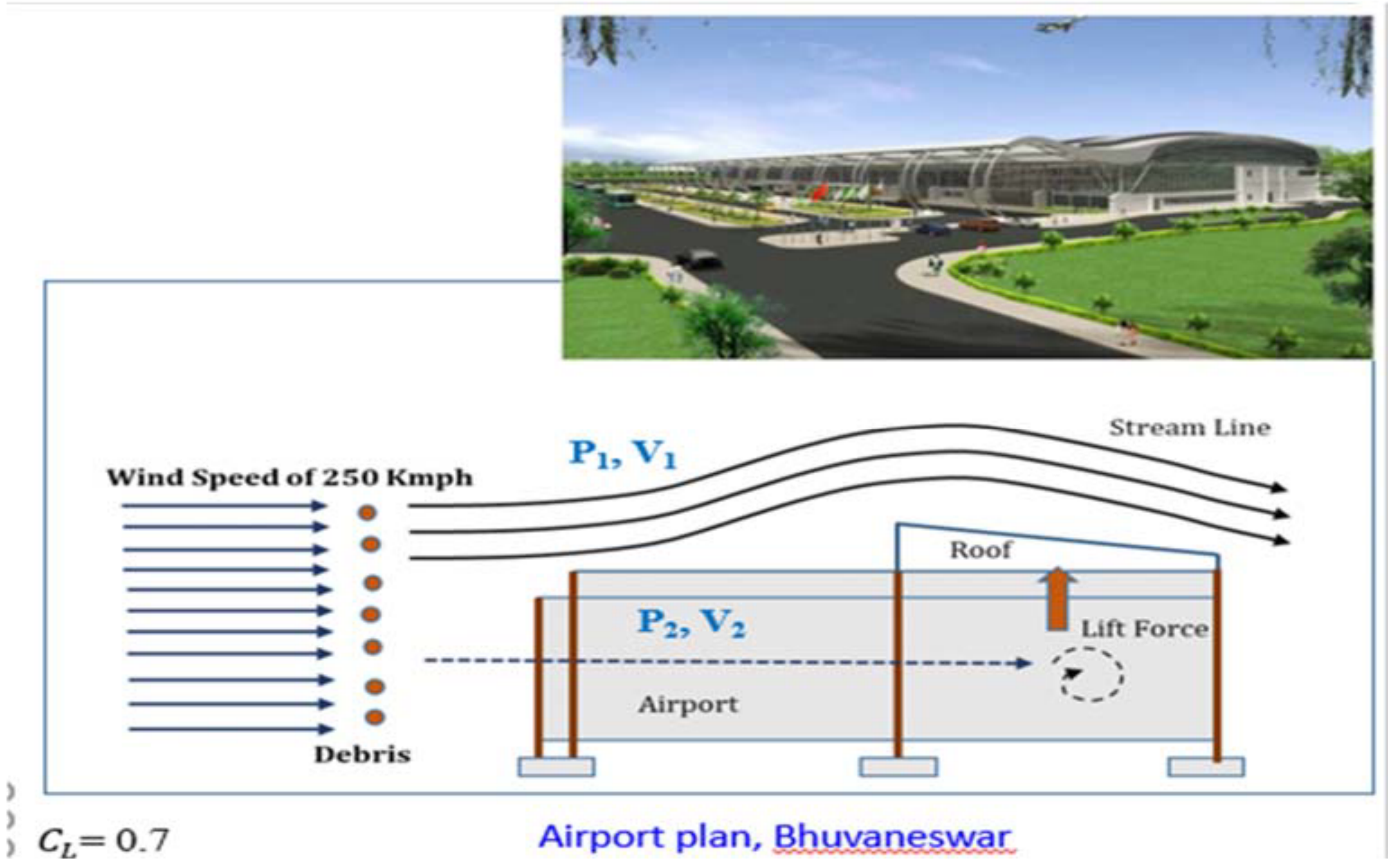


As we start with a real life example problems, today let us start with a real life problems like estimating the wind loads of a building where the cyclone effect is more disasters. The for example, if you look at this, the airport locations in Bhubaneswar and the cyclonic speed of the 250 kilometer per hours you can compute it what will be the pressure difference between the off stream in off area as well as the inside the airport.



And that pressure difference will uplift these the roof of this aircraft airport system. So the basically what I am to try to say that if you look at these problems which look it is very complex, but with help of the control volume concept and the drawing the streamlines we have then if you apply the Bernoulli's equations you can solve these problems to estimate what could be the wind loads when you have a cyclonic speed 250 kilometer per hours passing through this type of civil engineering structures.

P_1 = Atmospheric pressure at 50 m height

= 100725.8 N/m^2

$\frac{P_1}{\rho g}$ = 8381.76 m

V_1 = 250 Kmph (Design Wind Load)

= 69.44 m/s

$\frac{V_1^2}{2g}$ = 245.76 m

V_2 =175Kmph (Inside building)

= 48.60 m/s

$\frac{V_2^2}{2g}$ = 120.42 m

$\frac{P_2}{\rho g}$ = 8507.10 m

P_2 = 102232.02 N/m^2

Lift Force= $\frac{1}{2} C_L \rho A V^2$

= 25317.26 N = 25.3 KN

And if you look at this way, it is quite easy to compute the wind load estimations, but we should know exactly how to apply the Bernoulli's equations and what are the assumptions are there. How we can make a very complex problems in a simpler way by following the control volume concept, pressure diagrams, velocity diagrams, then applying the Bernoulli's equations.

So in this class, I will talk about three basic equations what we have derived in the last class. Those equations again I have to repeat it.

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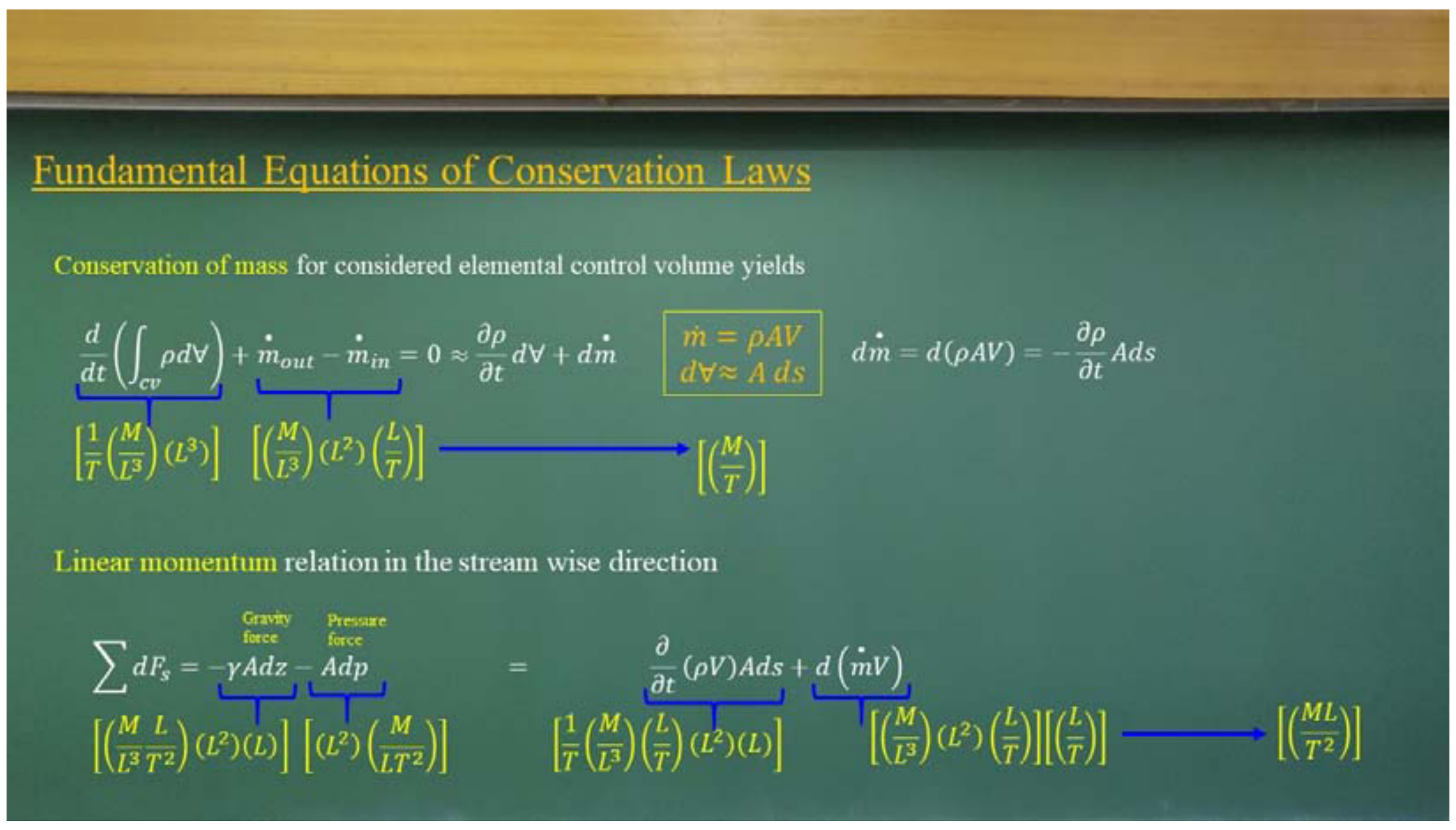

Like if you look it that these mass conservation equations which are very basic equations. If you look any control volume, we have considered is that this is a change in the mass flow rates. And this is mass out flux, influx. That is what is should equal to zero. This is what the mass conservation equations. But if you simplified that, it can come out to be Q 1 is equal to the Q 2.

Conservation of mass for considered elemental control volume yields

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) + \dot{m}_{out} - \dot{m}_{in} = 0 \approx \frac{\partial \rho}{\partial t} d\forall + d\dot{m}$$

$$\dot{m} = \rho AV$$

$$d\forall \approx A\,ds$$

$$d\dot{m} = d(\rho AV) = -\frac{\partial \rho}{\partial t} Ads$$

With there are certain assumptions like the steady flow, that one dimensional flow then mass inflow is equal to the mass outflow. Similar way if you look at this momentum equations what we derived earlier and that what will be expressions like this. So what you supposed to do it whenever write these type of big equations, try to write this all these dimensions and check it the dimensions of each components whether they are the equal.

Linear momentum relation in the stream wise direction

$$\sum dF_s = -\gamma Adz - Adp \;\; = \;\; \frac{\partial}{\partial t}(\rho V)Ads + d\left(\dot{m}V\right)$$

And from these dimensions you can understand it, what is the these component is indicating? Like for example for this case, it is a mass flux, mass per unit time. In this case, if you look it that, this is what the force components $\frac{ML}{T^2}$ is a force components. So each components representing as a force components. That is what we try to understand by writing the proper equations for conservation of mass and the linear moment.

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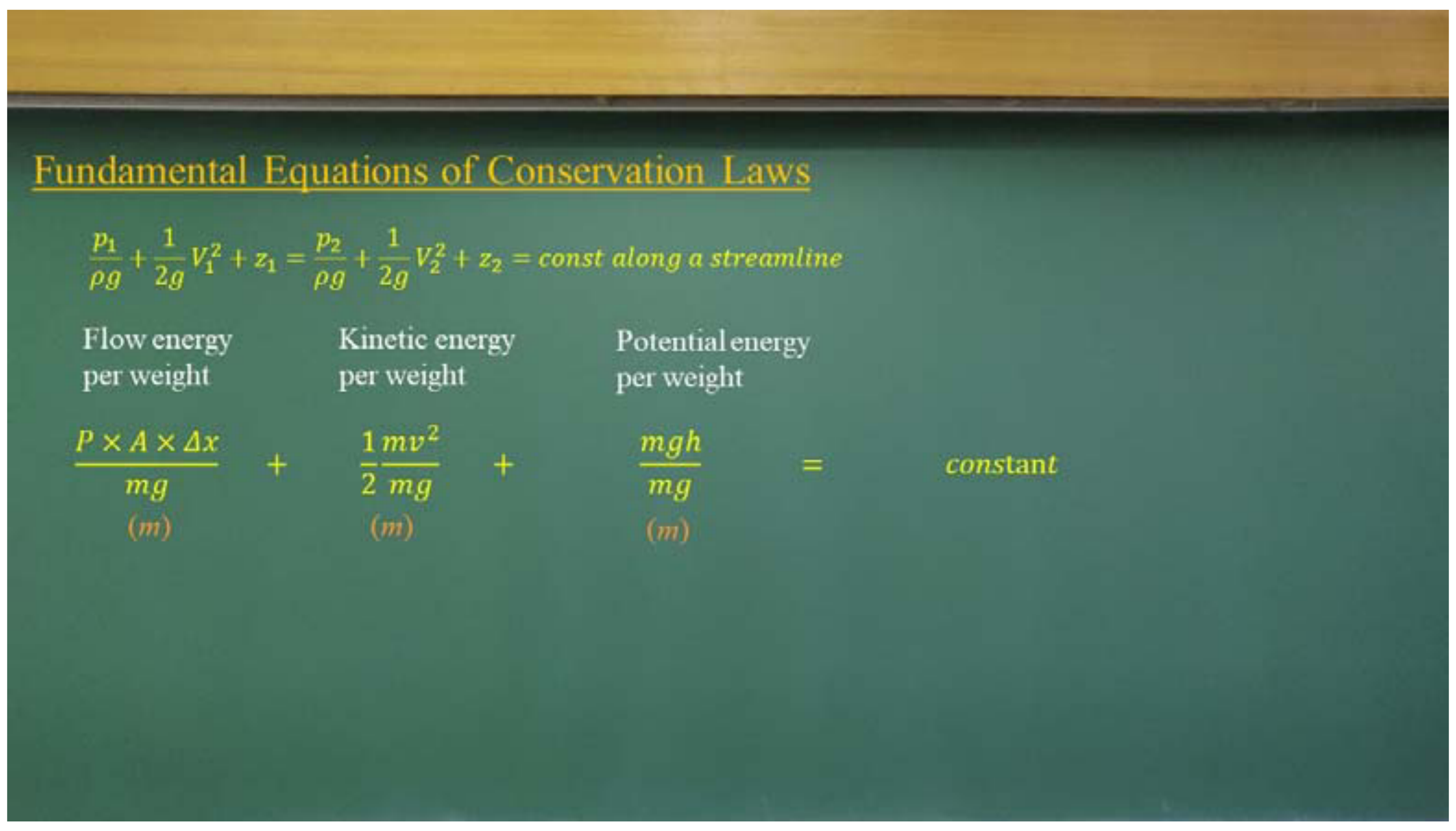

And if you look it that basic equations of Bernoulli's equations which has the pressure head component, the velocity head component and the elevations head those what if you include it as the flow energy for weight, the kinetic energy for weight and the potential energy for weight. That is what the unit weight is each one is in meters is equal to constants along a stream line. That is the basic assumptions what we have for the Bernoulli's equations.

$$\frac{p_1}{\rho g}+\frac{1}{2g}V_1^2+z_1=\frac{p_2}{\rho g}+\frac{1}{2g}V_2^2+z_2=const\ along\ a\ streamline$$

Flow energy per weight + Kinetic energy per weight + Potential energy per weight

$$\frac{P\times A\times \Delta x}{mg} \quad + \quad \frac{1}{2}\frac{mv^2}{mg} \quad + \quad \frac{mgh}{mg} \quad = \quad constant$$

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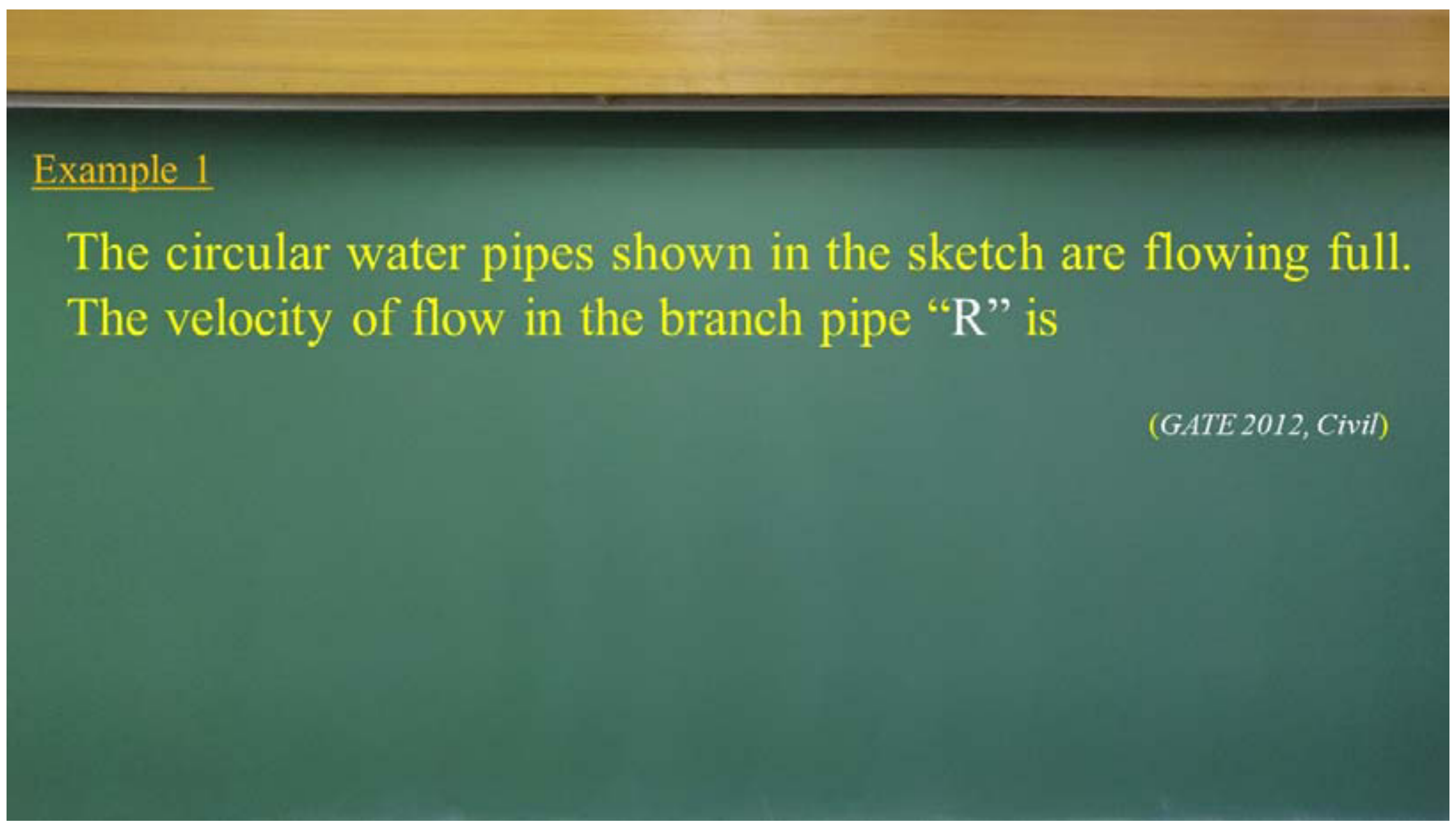

Let us solve the five questions from GATE question set. The first examples one what I consider is the circular water pipes shown in the sketch are flowing full condition. The velocity of the flow in the branch pipe R is that means let us sketch the figure first.

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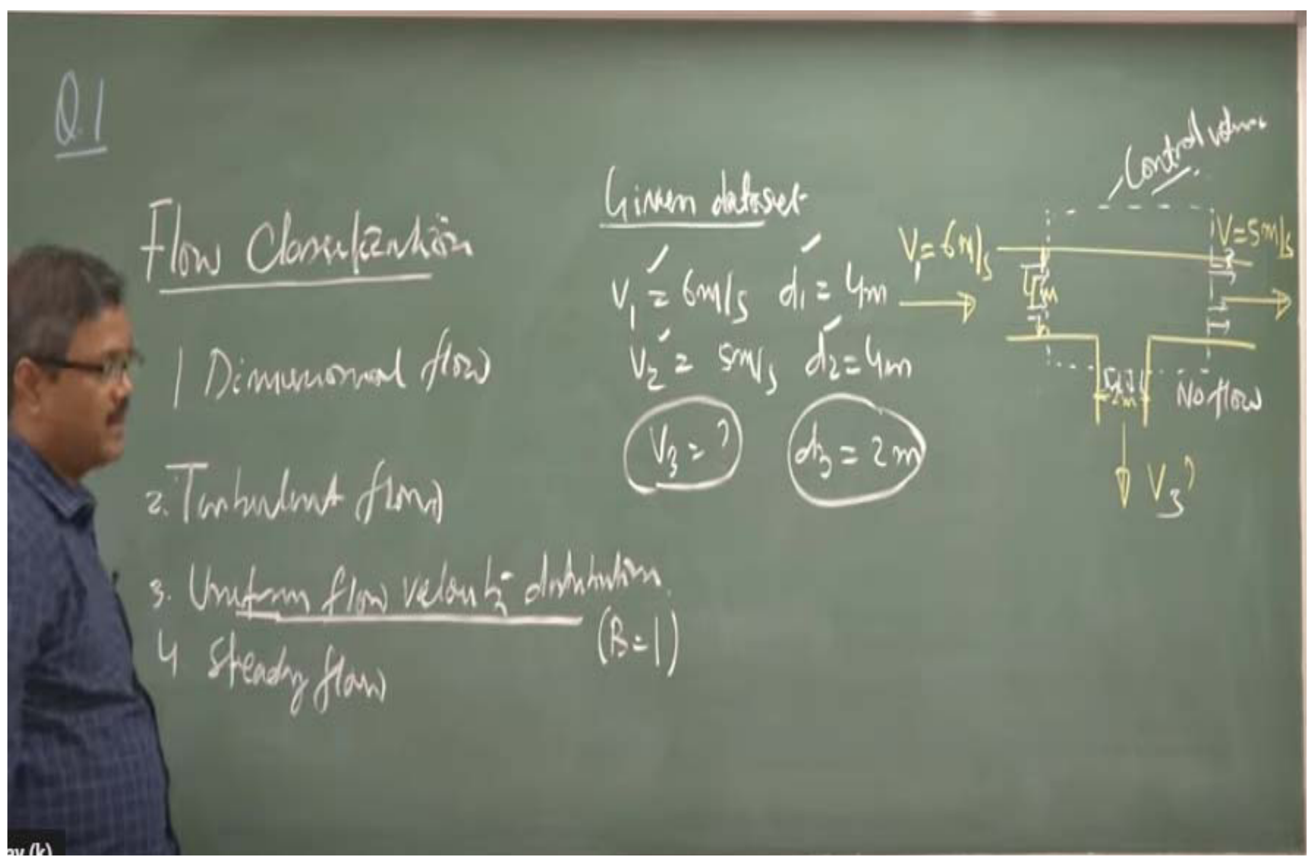

That there is pipe connections like this.

$$d_1 = 4m, \qquad d_2 = 4m, \qquad d_3 = 2m$$

$$V_1 = 6\,m/s\,, \qquad V_2 = 5\,m/s\,, \qquad V_1 = ?$$

This is what 4 meters and these dimensions is 2 meters and we have the velocity here V is equal to 6 meter per second and the velocity at this point V equal to 5 meter per second. And we need to compute what is the velocity at this point. So if you look at this problems, it is very simple problems of mass conservations. But before solving this problem, let us tell it the state wise.

After this sketching the problems, please draw a control volume, draw a control volume. And from this control volumes, you can identify this is what the control volumes, you can identify the inflow regions, the outflow regions where is no flow regions. We should identify it. So once you identify, sketch the control volume, sketch the control surface, identify the pressure the control surface and the flow directions.

Flow classification

- one dimensional flow
- turbulent flow
- flow distributions is uniform flow velocity distribution
- steady flow

In this case, we can have a the assumptions or the flow classifications is that first it is a one dimensional flow. Flow is the turbulent flow. And here we can consider the flow distributions is uniform flow velocity distributions. Once you have these assumptions, the problems is quite simplified for us. And the fourth one we can have the steady flow.

So please remember that whenever you draw the control volumes, you first classify the flow. First is one dimensional flow, turbulent, steady, it does not vary with the time and the uniform flow distributions. That means the beta is equal to 1. Then we will go for the what are the data is given. It is given data set. In this case, it is a very simplified case that if I find out the velocity and V 1 and V 2 and this is the V 3.

So I have the velocity V 1 is 6 meter per second, V 2 is equal to 5 meter per second and d 1 the diameter is here 4 meters; d 2 is also the 4 meters. But V 3 is unknown to us and d 3 is 2 meters. The problems if you look it, you can write the given data sets like this. So that from these control volume, you can find out what is the V 1, V 2, d 1, d 2. V 3 need to be computed and this d 3 is given to us.

Then we go for applying simple mass conservation equations. When you apply the mass conservation equations, you can start writing the mass conservation from basic RTT levels.

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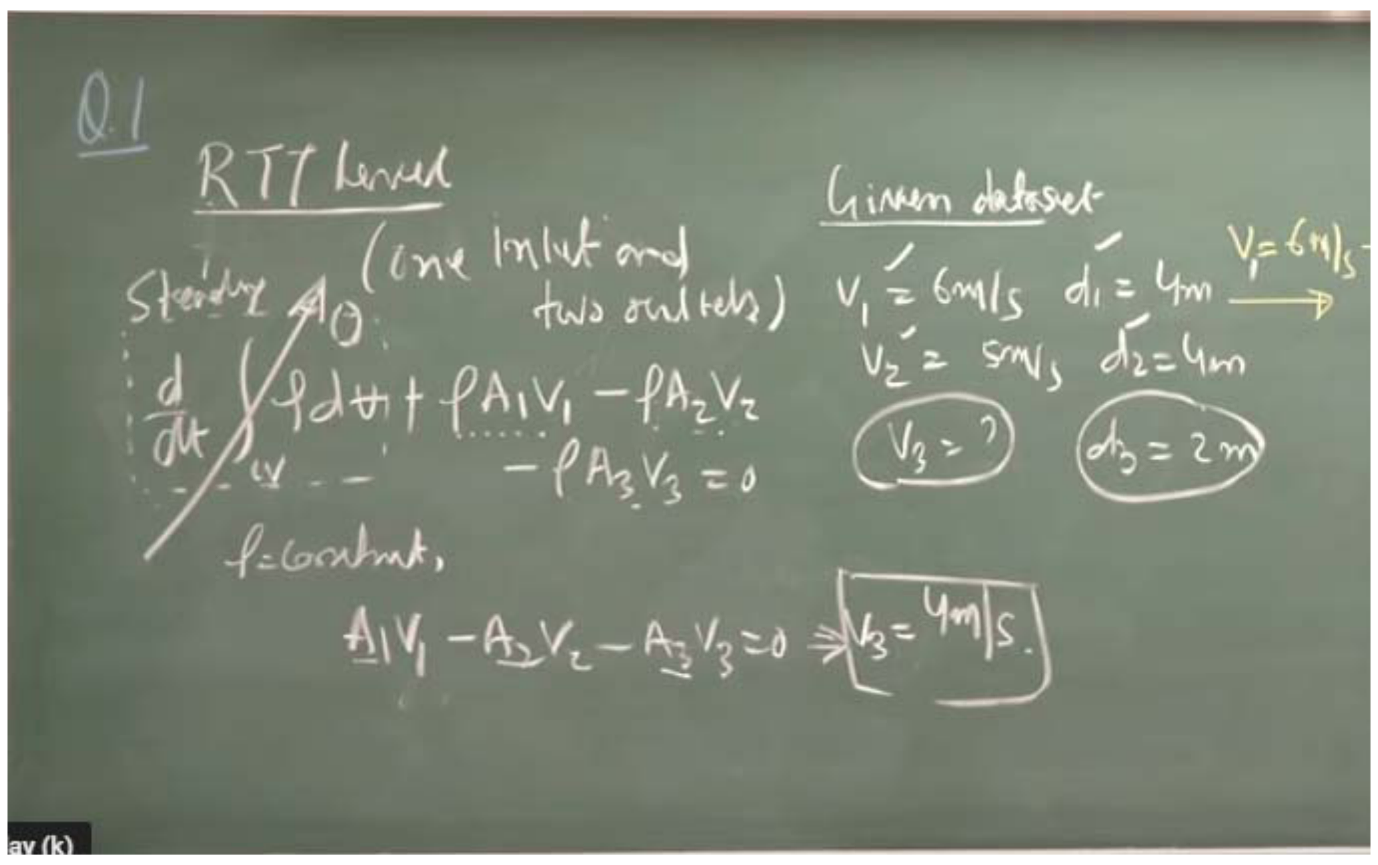

Given that

$$d_1 = 4m, \qquad d_2 = 4m, \qquad d_3 = 2m$$

$$V_1 = 6\,m/s\,, \qquad V_2 = 5\,m/s\,, \qquad V_1 = ?$$

Then you simplify the terms one by one, like if it is have a one inlet in this case and two outlets. So what we have that we have, we are just applying the mass conservation equations for this simplified the control volume concept. The mass conservation equations if you look it will have a okay, just you look it these equations. This is change it the mass storage within this control volume.

$$\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \rho A_1 V_1 - \rho A_2 V_2 - \rho A_3 V_3 = 0$$

$$\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall \text{ tends to 0, for steady flow.}$$

This is what the influx mass, these two are the outbox mass. So rho A into V. That is what the mass flux, rho A 2 and V 2. This is the mass flux outgoing in the A 2. Similar way we have rho A 3 V 3. So in this case, as it is the steady flow, as it is a steady flow, so these components becomes zero. So we have only these three components, what we need to compute it.

$$\rho = constant$$

$$A_1 V_1 - A_2 V_2 - A_3 V_3 = 0$$

$$V_3 = 4\, m/s$$

And as the rho is a constant or since it is a only water is flowing through this. So we can easily compute it that the A 1 V 1, A 2 V 2, A 3 V 3 is equal to zero. So you just substitute the velocities and the area of the flow at A1, A 2, A 3 we will compute the V 3. That is what is going to come 4 meter per second. So the numerical value I am not putting it individually, but please put it then you will get the solution V 3 is equal to 4 meter per second. More details we will provide in the slides.

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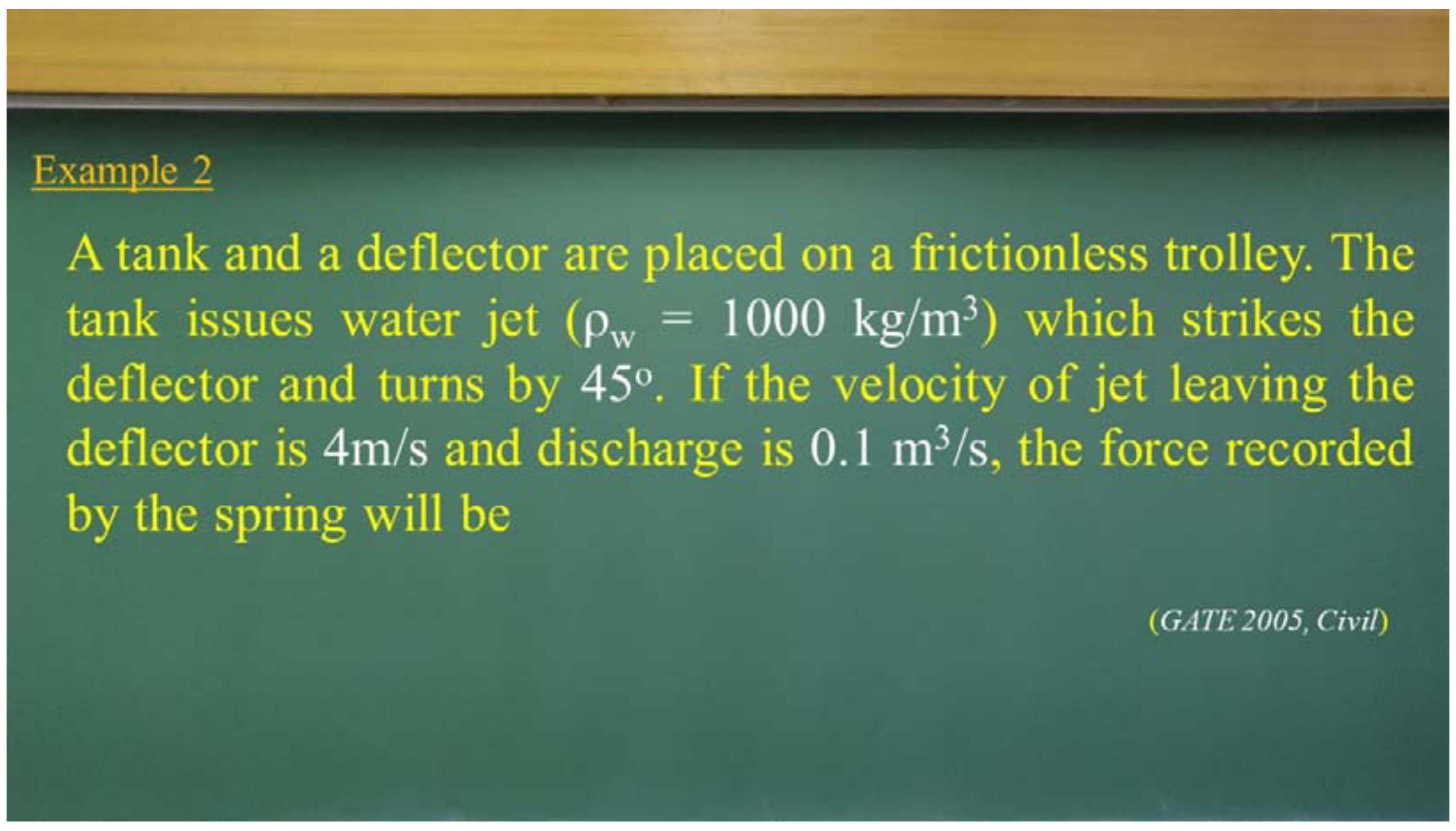

Let us start the question, example number 2 in which a tank deflectors are placed on a frictionless trolley. This is what frictionless trolley. That means there is no friction force components are there. The water issues a water jet.

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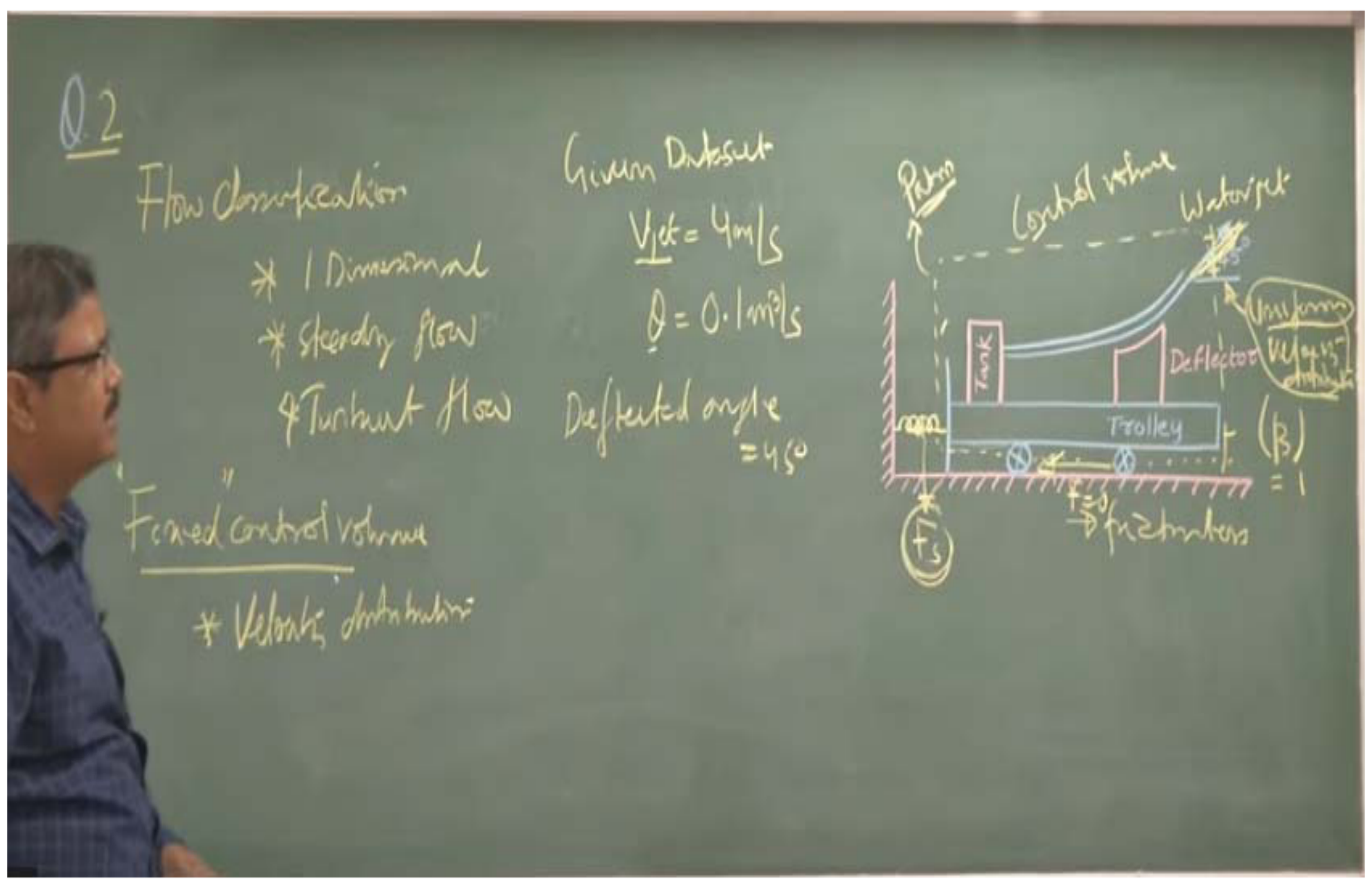

water jet which strikes the deflector turns 45° if the velocity of jet leaving the deflector is 4 meter per second, the V jet is equal to 4 meter per second and the discharge the Q is equal to 0.1 meter cube per second. Then we have to find out what could be the force acting on the spring. This is what the problems.

Given that,

Deflection angle= 45°

V_{jet} =4 m/s

Q = $0.1m^3/s$

If you look at these problems, even if you looks a complex things having the trolley, the water tank, the deflectors and the spring, but it is a very easy problems because we need to compute what will be the force acting on this spring. So first you sketch the problems then do the flow simplifications okay or the flow classifications. So in this case if you look it, we can assume it, it is a one dimensional force component as we sketch the control volumes like this okay.

So once I sketch this control volume like this, if you look it the technically there is no flow anywhere, the except there is a flow jet which is going out from this okay. That is what is happening this. The water jet is going out from this for this control volume. But there is a force component why it is acting on this the spring part which is the F x part. And there is no frictional forces.

So there is a force component at this is equal to the zero. So there is a no force components what is there what these the control surface and only this water jet is going out from this for this the control volume. Now we can simplify these problems as we classify the problems like this problems also we can say it is a one dimensional as we have the water jet problems. Secondly, have the steady flow.

It could be the turbulent the flow conditions. But what we have considered is a fixed control volume. We have considered fixed control volume and over these control surface everywhere the pressure is equal to P atmosphere. So that means we need not to consider the pressure diagram for this case as the pressure everywhere except this jet locations the pressure difference are there.

But other surface this pressure is equal to the atmospheric pressures. So looking this part as it is a fixed control volume we can write the basic equations. Before that as this flow distributions what is there the velocity distributions for this small jet what is going out from this we can consider as a uniform flow velocity distributions. As you consider it is a uniform velocity distributions the momentum correction factors beta becomes 1.
Flow classification

- one dimensional flow

- steady flow
- turbulent flow

So this is quite simplified and that is the conditions happens when you have a water jet is going out there could be a velocity distribution there. But the assumption of uniform velocity distribution also quite valid for this type of calculations. Now let us have the data given a given data set. One is the velocity, jet velocity, the discharge, the Q value. Second is that the deflected angle. In this case it is 45 degree, okay.

So in this case if you look at this problems is quite simplified problem, because we need not to apply any mass conservation equations because we just need it what could be the force acting on the spring. That means it is quite easy problems. Just we have to apply linear momentum equations for these control volume. Let us write the linear momentum equations for these control volumes. Then we will simplify the linear momentum equations to get the solutions for this.

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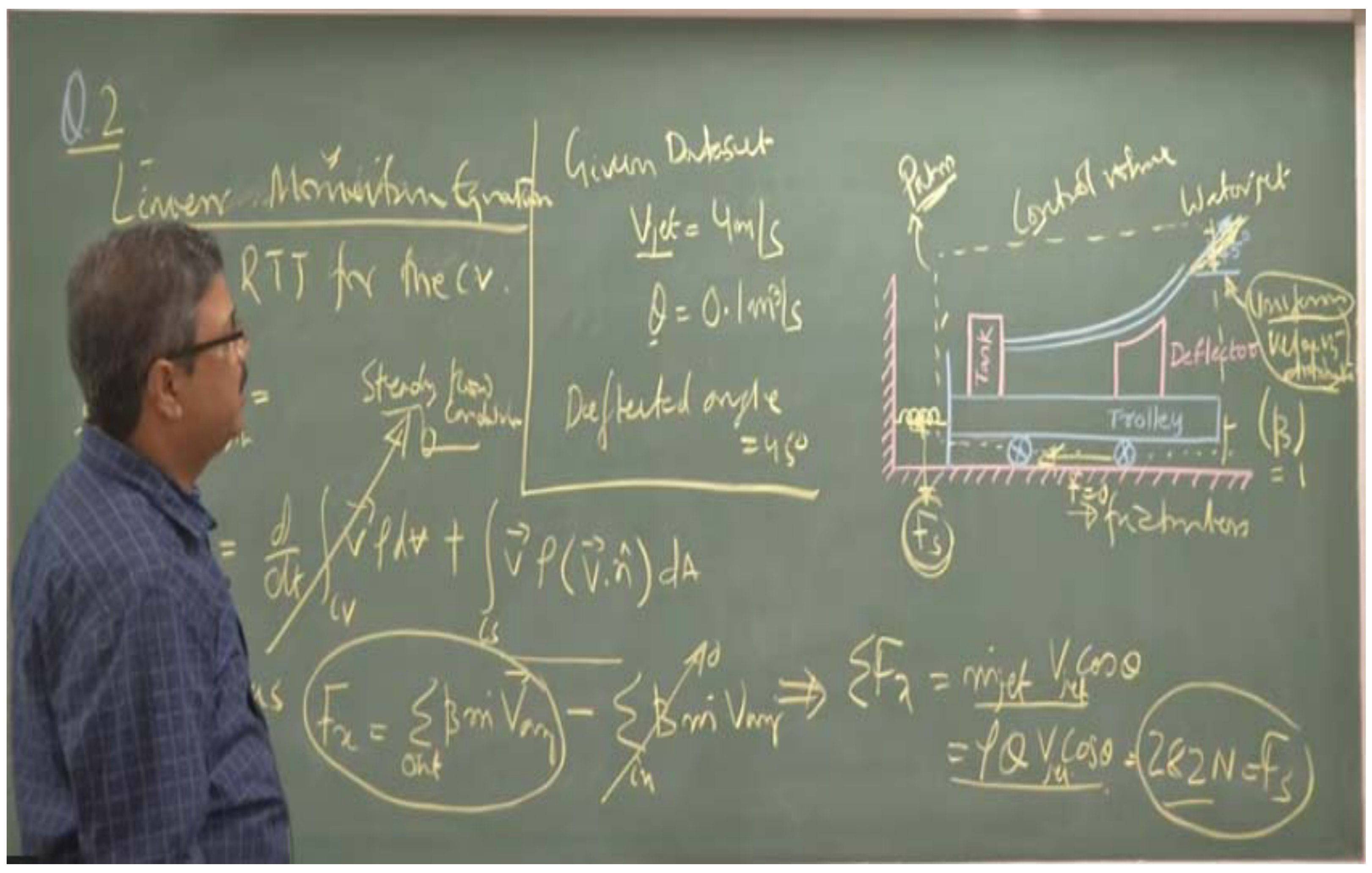

So linear momentum equations. If I write it from the applying Reynolds transport theorems for the control volume, we will get it that d/dt of system that is what equal to the sum of the force acting on this will be control volume control surface. This is what if you can remember it the Reynolds transport theorems the sum of the force acting on this control volume set is equal to the rate of change of the momentum flux within the control volume.

$$\sum \vec{F} = \frac{d}{dt}\left(\int_{cv} \vec{V}\rho d\forall\right) + \int_{cs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

Net of the change of the momentum flux passing through these controls surface, the net momentum flux, that is what will be equate for this case. So if you look at this case, this is becomes zero as if the steady flow conditions. This component become zero. So only you have a sum of the force components and the momentum flux through this control surface. That is what we need to put it.

So in this case if I put the along the x axis, along the x axis if I write these equations, then F x will be equal to sum of momentum flux out beta m dot V average minus beta m dot V average as in. So in this case as the beta becomes 1, the mass flux in and out. So in this case, there is no influx. That is what this is equal to zero and the force components will be the only this component, only this component.

$$\sum \overrightarrow{F_x} = \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

$$\sum \overrightarrow{F_x} = \dot{m}_{jet} V_{jet} \cos\theta$$

$$= \rho Q V_{jet} \cos\theta = 282N$$

So this way we can solve these problems.

Let me highlight that way that first you draw the control volume. Once you draw the control volume, try to find out what could be the pressure distributions. Whether we can neglect there is a pressure difference at the water jet part. And then you look at what are the mass influx and outflux is going out or momentum flux going in or coming out from the control surface.

Once you conceptually start, then you apply. In this case, we just apply the linear momentum equations because mass conservation equations is not required for this case and using this the appropriate assumptions we simplify this Reynolds transport equation to very simplified the force component along the x directions and these force component will be equated with the force component acting on this.

That means the force acting at the spring, that is what will be come it is this value. So you can use linear momentum equations to solve this type of the problem.

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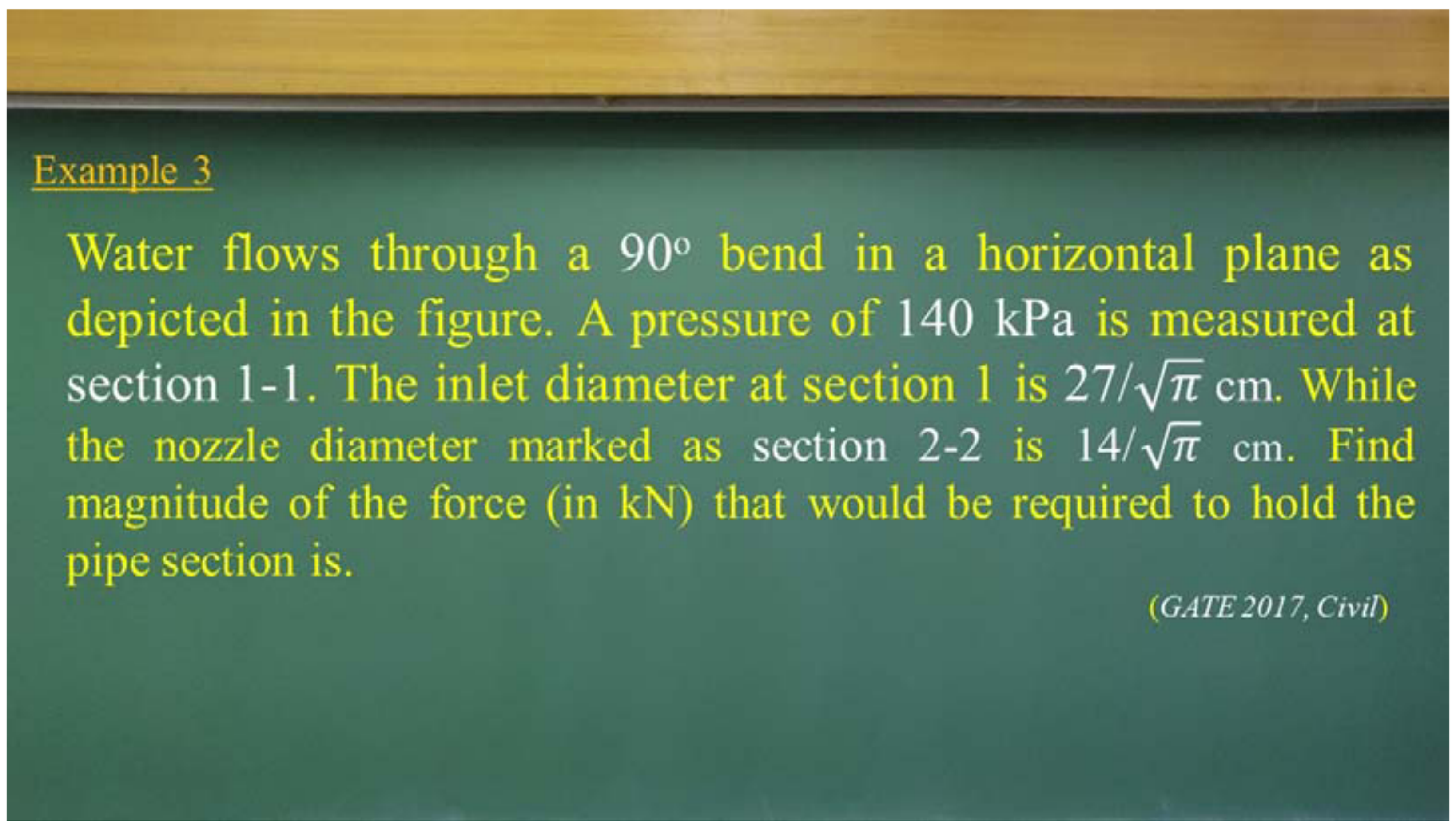

Let us go for the example 3 which is about a water flow through a 90 degree bend in a horizontal plane as given in the figures. The pressure at 140 kilo Pascal is measured at the section 1-1. The inlet diameter the section is about 27 by square root of 5 centimeters, where the nozzle diameter marked as section 2-2 which is 14 divide by square root of the 5 centimeter. Find the magnitude of the force in kilo Newton that could be required to hold this pipe sections.

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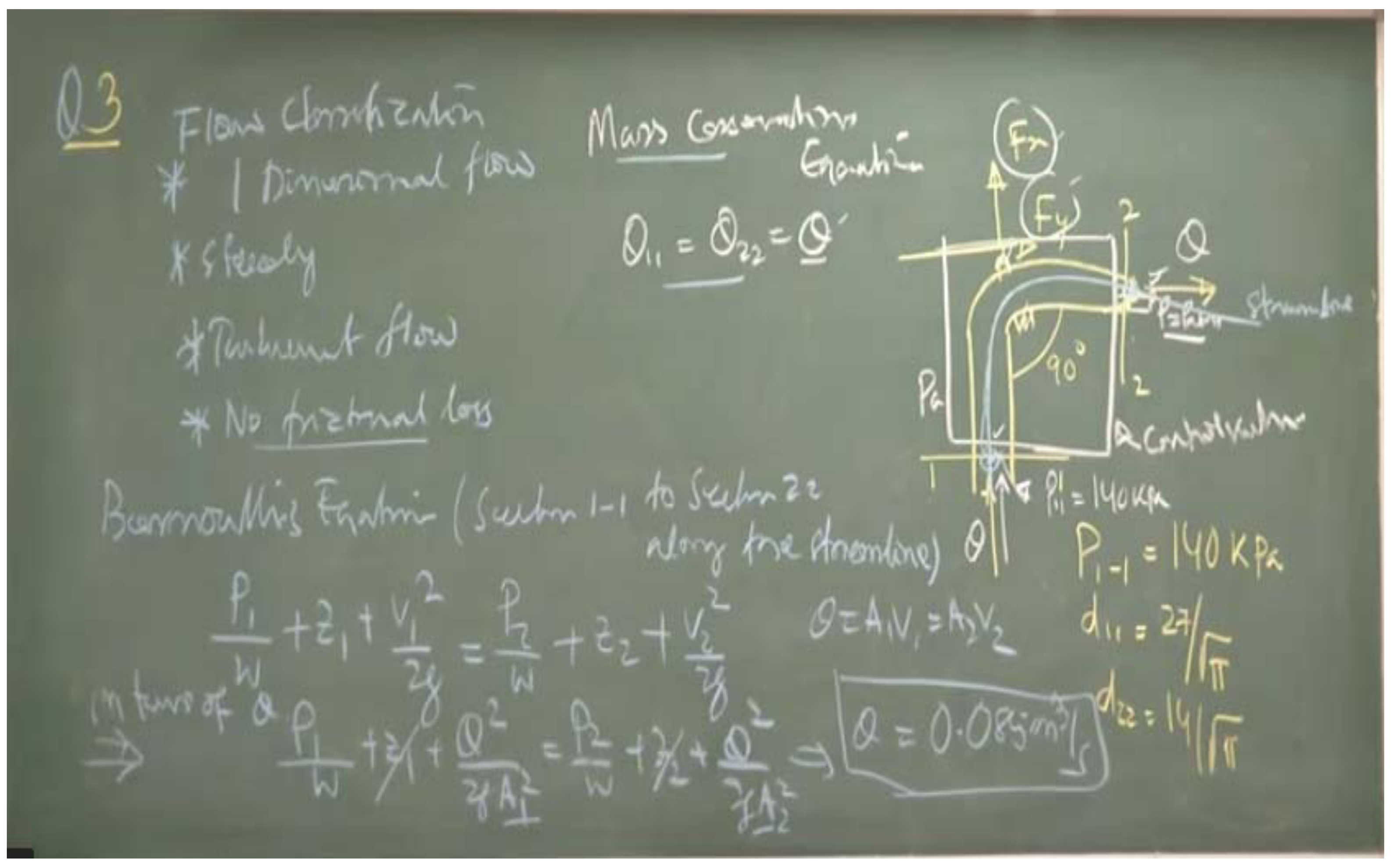

So if is that problem, let us first sketch this problem. The problem is there, there is nozzles and this making like this surface. So it has sections 1 and sections 2-2. This is inlet, this is the outlet. The pressure at the inlet 1-1 is given it 140 kPa. Pressure at 1-1 is given as this and

the pipe inlet diameter d 1-1 is given as

$$d_{11} = \frac{27}{\sqrt{\pi}}$$

d 2 section 2-2 it is given as

$$d_{22} = \frac{14}{\sqrt{\pi}}$$

So the question is that the how much force would be required to hold this pipe, okay. So if you can understand from these problems, as the flow is going here and we are deflecting a 90˚bend, this is what the 90˚bend there will be a change in the momentum flux, the direction.

That is what will be exerted force on these the pipe. So if you look at these problems, it is a quite easy problems to find out what will be the force components as if I give a two directions is F_y and F_x what will be the force component acting on this the 90˚bend part. So first what you have to do it you draw a control volume, draw a control volume and you have a external force acting on this is F_x and F_y.

And here you have everywhere your pressure equal to atmospheres. At this point you have pressure is equal to atmospheres. But at this inlet point you have a pressure which is 140 kPa. So know the pressure diagrams, you know the pressure diagrams. You know this control volume and here you have the velocity the flow velocity is there and here also you have a the flow velocity component is there.

If you look at this control volumes, here you can easily know it because there is one inlet and one outlets. You can define that Q amount of this set if it come into this and going out the same Q will come it here. So the problems is here the Q_{11} is equal to Q_{22}, that is what will be Q. This is what the mass conservation equations which is indirectly implemented in this case as there is a one inlet and one outlet case.

So Q is constant here. So if you have a Q is a constant, I can draw a streamlines, I can draw a streaming here. I can draw a streamline and along the streamlines at these two points at 1-1 section and the 2-2 sections, we can apply the Bernoulli's equations for this case. So the assumptions what we have done it or the flow classifications, flow is one dimensional flow, steady, turbulent.

And we have considered no frictional loss or we have neglected the frictional losses what will be there because the pipe water flow through this nozzles that is what we are consider. There is no frictional losses or frictional loss is not that dominated as compared to the force component what is acting on this. So if you have this case, so first you have a mass conservation equation you have applied it here.

$$\frac{p_1}{w} + \frac{1}{2g}V_1^2 + z_1 = \frac{p_2}{w} + \frac{1}{2g}V_2^2 + z_2$$

Now I am going to apply to compute what could be the Q value we are applying the Bernoulli's equations. We are applying Bernoulli's equations as I have drawn the streamlines following through this path. I can just apply the Bernoulli's equations at the section 1-1 and the sections 2-2, section 1-1 to section 2-2 along the streamline. If I apply that then what I will write it P is equal to P 2.

$$Q = A_1V_1 = A_2V_2$$

$$\frac{p_1}{w} + \frac{Q^2}{2gA_1^2} + z_1 = \frac{p_2}{w} + \frac{Q^2}{2gA_2^2} + z_2$$

$$Q = 0.085\, m^3/s$$

That is what we applied it. Since it is on the horizontal surface, you can consider $z_1 = z_2$ to equal to z value. That what will cancel it. We know this the pressure values here. We have to compute now.

We know this A_1 value A_2 value. We have to compute what will be the Q value. From these substituting the pressures and the substituting the A_1, A_2 value, we can compute it the Q values here. That is what will come out to be, the Q will become 0.085 m^3/s. So now if you just look it, we have applied the Bernoulli's equations for these problems first to compute what will be the discharge.